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Duality for massive spin two theories in arbitrary dimensions

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Abstract: Using the parent Lagrangian approach we construct a dual formulation, in the sense originally proposed by Curtright and Freund, of a massive spin two Fierz-Pauli theory in arbitrary dimensions D. This is achieved in terms of a mixed symmetry tensor $T_{A[B_1B_2\ldots B_{D-2}]}$, without the need of auxiliary fields. The relation of this method with an alternative formulation based on a gauge symmetry principle proposed by Zinoviev is elucidated. We show that the latter formulation in four dimensions, with a given gauge fixing together with a definite sequence of auxiliary fields elimination via their equations of motion, leads to the parent Lagrangian already considered by West completed by a Fierz-Pauli mass term, which in turns yields the Curtright-Freund action. This motivates our generalization to arbitrary dimensions leading to the corresponding extension of the four dimensional result. We identify the transverse true degrees of freedom of the dual theory and verify that their number is in accordance with those of the massive Fierz-Pauli field.

Keywords: Duality in Gauge Field Theories, Gauge Symmetry.

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Contents

1. Introduction

Fields with spin two and higher in dimensions larger than four are of considerable interest for understanding supersymmetric string theories together with their brane extensions from the perspective of the M-theory. An additional feature that adds interest to this problem is that in dimensions $D > 5$, the totally symmetric tensor fields are not enough to cover all the irreducible representations of the Poincaré group. Thus, when dealing with higher spin theories it becomes necessary to take into account fields with mixed symmetry [1−3] belonging to "exotic" representations of the Poincaré group. It is therefore quite natural to expect that, in a field theory limit, the superstring theory should reduce to a consistent interacting supersymmetric theory of massless and massive higher spin fields. In four space-time dimensions, Lagrangian formulations for massive fields of arbitrary spin were originally constructed in ref. [4]. Later, this construction was used to derive Lagrangian formulations for massless gauge fields of arbitrary spin [5]. An important matter related to mixed symmetry tensor fields is the study of their consistent interactions, among themselves as well as with higher-spin gauge theories [6]. Amid the many approaches to the problem, a particularly interesting one is the Zinoviev approach [7] where the gauge symmetry principle has been extended to deal with the massive case in a way that incorporates a Stueckelberg-like formulation of the corresponding actions in the background of Minkowski and (A)dS spaces.

The proliferation of "exotic" mixed symmetry fields poses the question of identifying different representations that can describe the same spin, possibly in different phases with respect to a weak/strong coupling limit. This is precisely the subject of duality, which has been profusely studied along the years in many different contexts [8, 9]. In the massless case, dual formulations of fields with spin two and higher in arbitrary dimensions have been derived from a first order parent action [10] based upon the Vasiliev action [11]. In this case, when the original description of the gauge fields in dimension D is in terms of totally symmetric tensors, dual theories in terms of mixed symmetry tensors corresponding to Young tableaux having one column with $(D-3)$ boxes plus $(s-1)$ columns with one box have been obtained [10]. A discussion of duality for massless spin two fields in arbitrary dimensions, which is consistent with the Vasiliev formulation [11], has been presented in ref. [12]. An alternative construction of theories which are dual to linearized gravity in arbitrary dimensions has been developed in ref. [13], following the method of the global shift symmetry applied to the tetrad field.

Contrary to the massless case, dual formulations for massive gravity are not as well explored and still present issues requiring elucidation. The basic idea of dualizing the massive Fierz-Pauli (FP) action [14], written in terms of the standard symmetric tensor $h_{\alpha\beta}$, is to find a formulation where the kinetic contribution of FP yields the mass term contribution of the corresponding dual theory, and vice-versa.[1] There are many ways, not necessarily equivalent, to construct dual theories. A convenient tool to achieve this is through the use of a first order parent action which contains both fields and which produces the dual theories via the elimination of the adequate field using its equations of motion [15].

Curtright proposed a dual description of the massive FP action based upon the mixed symmetry tensor $T_{A[BC]}$ satisfying the same identities as the linearized spin connection of Einstein's theory in arbitrary dimensions [1]. The corresponding kinetic term was constructed by imposing gauge invariance under general gauge transformations that respect these identities, which completely fixed the corresponding relative coefficients. The mass term was chosen to provide the standard energy-momentum relations for massive fields. In ref. [2], Curtright and Freund (CF) tried different parent actions in four dimensions to obtain the duality transformation between the FP action and that corresponding to the mixed symmetry tensor, but they were not able to obtain such a connection. They could only construct parent actions where the $h_{\alpha\beta}$ field satisfied the FP action, but the mixed symmetry tensor $T_{\alpha[\beta\gamma]}$ was associated to an action which was different from the one dictated by the gauge symmetry requirements imposed by their construction. Anyway, the impossibility of obtaining a dualization of massive FP was not conclusively proved, and they remarked the necessity of a definitive analysis of the subject.

Motivated by such results, a constructive method based on the parent Lagrangian approach was pursued in refs. [16, 17], which dispensed from the gauge invariance requirements of the action dual to FP. The starting point of the procedure is a second order Lagrangian in four dimensions, which depends on the fields φ^a and their derivatives $\partial_\mu\varphi^a$.

[1]The indices of D-dimensional tensors will be denoted with capital latin letters, while greek letters will be specifically used for the 4-dimensional ones. Square brackets will indicate antisymmetrization and curly brackets symmetrization. The Minkowski metric is denoted by $\eta_{AB} = \eta_{BA} = diag(-1, +1, \cdots + 1)$ and the completely antismmetric tensor by $\epsilon_{A_1A_2\ldots A_D}$. Only in these cases the symmetry is not explicitly stated by the corresponding brackets.

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As the first step, a first order Lagrangian is constructed using a generalization of a procedure presented in ref. [18], by introducing, via appropriate Lagrange multipliers L_a^μ, an adequate number of invertible auxiliary variables $f_\mu^a = f_\mu^a(\varphi^a, \partial_\mu \varphi^a)$. This intermediate Lagrangian contains the fields f_μ^a only in algebraic form, and thus they can be eliminated from the corresponding equations of motion. The resulting Lagrangian contains the derivatives of the original fields φ^a times the Lagrange multipliers L_a^μ, which become auxiliary variables. A point transformation in the extended configuration space for the auxiliary variables L_a^μ, $L_a^\mu = \epsilon^{\mu\nu\sigma\tau} H_{a\nu\sigma\tau}$, incorporates the intuitive idea of duality and yields the parent Lagrangian from which both dual theories can be obtained. The equations of motion for $H_{a\nu\sigma\tau}$ make these fields explicit functions of φ^b, $H_{a\nu\sigma\tau}(\varphi^b)$, and allows to go back to the original action after they are substituted in the parent Lagrangian. On the other hand we can also eliminate the fields φ^a from the parent Lagrangian using their own equations of motion, and in such a way we obtain a new theory that only contains the $H_{a\nu\sigma\tau}$. This new Lagrangian is dual to the original one, and the equivalence is given by the transformations defined by the equations of motion of the parent Lagrangian. This approach gives a parent Lagrangian with a minimum number of fields: the original ones and their duals. The generalization of this approach to higher order Lagrangians as well as to arbitrary dimensions is straightforward.

Applying this procedure to the massive spin two field $h_{\{\mu\nu\}}$, we started with the standard Fierz-Pauli Lagrangian and constructed a family of first order Lagrangians that contain the symmetric tensor $h_{\{\mu\nu\}}$ and the mixed symmetry tensor $T_{\alpha[\mu\nu]}$. Using the equations of motion for $h_{\{\mu\nu\}}$ we can eliminate this last field, in such a way that we obtain a set of multiparametric families of Lagrangians dual to massive Fierz-Pauli, where the dynamics is now contained in the $T_{\alpha[\mu\nu]}$ field. The unique kinetic term of these dual actions is fixed by the choice of the FP mass term in the parent action, while only the corresponding mass terms incorporate the free parameters. A practical approach to obtain the general structure of such parent Lagrangians amounts to writing the most general combinations of terms quadratic in the dual fields $h_{\{\mu\nu\}}$ and $T_{\alpha[\beta\gamma]}$, plus a combination of all the possible coupling terms which are linear in each of the dual fields and include one derivative. The arbitrary coefficients are partially fixed by eliminating $T_{\alpha[\beta\gamma]}$ from the parent Lagrangian and demanding the recovery of the Fierz-Pauli action. After a careful inspection of all dual Lagrangians obtained with this method it becomes clear that the Curtright Lagrangian is not obtained.

A key to understand this difficulty is given by the first order action proposed by West [12]. It has the form of a Lagrangian in our multiparametric family, but with $h_{\mu\nu}$ without a definite symmetry, instead of the symmetric one chosen in [16, 17]. When $T_{\alpha[\mu\nu]}$ is eliminated from this first order Lagrangian, the antisymmetric part of $h_{\mu\nu}$ decouples and becomes irrelevant, so that we obtain the usual massive Fierz-Pauli theory. On the other hand, when we eliminate the $h_{\mu\nu}$ field, the presence of its antisymmetric part alters the dynamics of $T_{\alpha[\mu\nu]}$, thus extending the families of dual Lagrangians for the massive spin two field to include the form proposed by Curtright.

A different approach was followed by Zinoviev [19] based on a Stueckelberg-like construction for massive tensor fields in Minkoswki as well as (Anti) de Sitter spaces. For the spin two case in four dimensions he starts from a first order parent action incorporating the fields ($\omega_{\mu[\alpha\beta]}$, $F^{[\alpha\beta]}$, π^α) which are dual to ($h_{\mu\alpha}$, A_μ, φ). The massive first order parent action is constructed from the corresponding free actions for the massless version of the above mentioned fields, plus additional mass terms which induce a redefinition of the original gauge transformations for the massless fields in order to preserve a mass dependent gauge invariance of the full action. In this sense ($F^{[\alpha\beta]}$, π^α) together with (A_μ, φ) are the auxiliary Stueckelberg fields for the resulting spin two massive dual fields $h_{\mu\alpha}$ and $\omega_{\mu[\alpha\beta]}$. The construction is presented in four dimensions and no general prescription for arbitrary dimensions is given, as it has been done for example in the massless case in ref. [10], except for the statement that the method can be easily generalized in such a case.

The paper is organized as follows. In section II we demonstrate the equivalence between the dual Zinoviev action, with an adequate gauge fixing, and the Curtright-Freund action in four dimensions. We also start from the Zinoviev parent action (which includes a non-symmetrical $h_{\mu\nu}$) and show that the elimination of some auxiliary fields together with additional gauge fixing leads to the first order parent action proposed by West [12] plus a FP mass term. From this parent action we recover, on one hand, the Fierz-Pauli formulation in terms of the symmetric part of $h_{\mu\nu}$ and, on the other, the Curtright-Freund dual theory in terms of the field $T_{\alpha[\beta\gamma]}$. This duality, described in section III, directly relates the description in terms of a symmetric FP field h_{AB} and a mixed symmetry tensor $T_{B[A_1 \dots A_{D-2}]}$, satisfying the cyclic identity, in arbitrary dimensions D. This construction does not require the use of additional Stueckelberg-like fields. The count of the true degrees of freedom of the dual field $T_{B[A_1 \dots A_{D-2}]}$ is also performed in this section. The last section contains a summary and comments on the work. Finally, we conclude with two appendices. In appendix A we include some useful expressions incorporating properties of the antisymmetrized generalized delta function which have proved useful in the calculations. Appendix B contains the derivation of the Lagrangian constraints satisfied by the dual field $T_{B[A_1 \dots A_{D-2}]}$ that arise from the corresponding equations of motion and which are required in section III to obtain the correct number or propagating degrees of freedom.

2. The Zinoviev approach and the Curtright-Freund formulation in a four dimensional Minkowski space

It is relevant to understand the relation between the Zinoviev approach, based on a first order parent Lagrangian having well defined gauge symmetries generated by a set of auxiliary fields [19], and the scheme proposed in ref. [16], based on the most general form for the first order parent Lagrangian containing only the dual fields. In the approach of ref. [19] a duality transformation between Stueckelberg-like Lagrangians for massive fields is obtained, while in that of refs. [16, 17] the duality is directly stated at the level of the fields $h_{\alpha\beta}$ and $T^{\rho[\mu\nu]}$ corresponding to different representations for the massive spin two degrees of freedom. When comparing with works of Zinoviev one has to keep in mind that his metric is $diag(+,-,-,-)$ so that we will need to make the appropriate changes of signs to translate his results into those corresponding to our choice of the metric. Let us recall that a consistent way of getting the correct relative signs is to count the total number of

$\eta_{\alpha\beta}$ factors in a given expression, each of which carries a minus sign. Sometimes we make a global change of sign in the resulting transformed Lagrangian.

2.1 The Curtright-Freund action from the Zinoviev dual action

A closer look at the final result of ref. [19] for the dual action to FP in terms of the field $\omega^{\mu[\alpha\beta]}$ reveals the notable feature that, after gauge fixing, it is equivalent to the CF action in terms of the field $T^{\rho[\mu\nu]}$, which is the Hodge dual of $\omega^{\mu[\alpha\beta]}$. To show this property in a simple way let us start from eq. (2.12) of ref. [19] which we rewrite here in flat space (zero cosmological constant) and in the metric $(-,+,+,+)$

$$\mathcal{L}_Z = \frac{1}{2}\left(R^{\mu\nu}R_{\nu\mu} - \frac{1}{3}R^2\right) + \frac{1}{6}\left(\partial^\alpha F_{[\alpha\beta]}\right)^2 + \frac{m}{\sqrt{2}}\left(\omega^{\mu[\nu\alpha]}\partial_\alpha F_{[\mu\nu]} + \frac{1}{3}\omega^\mu\partial^\alpha F_{[\alpha\mu]}\right)$$
$$+\frac{m^2}{2}\left(\omega^{\mu[\alpha\beta]}\omega_{\alpha[\mu\beta]} - \frac{1}{3}\omega^\mu\omega_\mu\right), \tag{2.1}$$

with

$$\omega^\mu = \omega_\alpha{}^{[\alpha\mu]}, \;\; R_{[\mu\nu][\alpha\beta]} = \partial_\mu\omega_{\nu[\alpha\beta]} - \partial_\nu\omega_{\mu[\alpha\beta]}, \;\; R_\mu{}^\nu = R_{[\mu\alpha]}{}^{[\alpha\nu]}, \;\; R_{\mu\nu} \neq R_{\nu\mu}, \tag{2.2}$$

which is gauge invariant under the transformations

$$\delta\omega_\mu{}^{[\alpha\beta]} = \partial_\mu\theta^{[\alpha\beta]},$$
$$\delta F^{[\mu\nu]} = -m\sqrt{2}\theta^{[\mu\nu]}, \tag{2.3}$$
$$\delta\omega^{\mu[\alpha\beta]} = \eta^{\mu\alpha}\xi^\beta - \eta^{\mu\beta}\xi^\alpha. \tag{2.4}$$

We can use the gauge freedom associated to $\theta^{[\mu\nu]}$ to set $F^{[\mu\nu]} = 0$, which leave us with

$$\mathcal{L}_Z = \frac{1}{2}\left(R^{\mu\nu}R_{\nu\mu} - \frac{1}{3}R^2\right) + \frac{m^2}{2}\left(\omega^{\mu[\alpha\beta]}\omega_{\alpha[\mu\beta]} - \frac{1}{3}\omega^\mu\omega_\mu\right), \tag{2.5}$$

which is still invariant under the transformations (2.4). After writing the kinetic part in terms of $\omega^{\mu[\alpha\beta]}$, the above Lagrangian reduces to

$$\mathcal{L}_Z = \frac{1}{2}\partial^\beta\omega_{\mu[\alpha\beta]}\partial_\theta\omega^{\alpha[\mu\theta]} - \frac{1}{6}\left(\partial_\alpha\omega^\alpha\right)^2 + \frac{m^2}{2}\left(\omega^{\mu[\alpha\beta]}\omega_{\alpha[\mu\beta]} - \frac{1}{3}\omega^\mu\omega_\mu\right). \tag{2.6}$$

It is convenient to split $\omega^{\mu[\alpha\beta]}$ into a traceless piece $\bar{\omega}^{\mu[\alpha\beta]}$ and the trace ω^β,

$$\omega^{\mu[\alpha\beta]} = \bar{\omega}^{\mu[\alpha\beta]} + \frac{1}{3}\left(\eta^{\mu\alpha}\omega^\beta - \eta^{\mu\beta}\omega^\alpha\right), \;\; \bar{\omega}_\alpha{}^{[\alpha\beta]} = 0, \tag{2.7}$$

which transforms as

$$\delta\bar{\omega}^{\mu[\alpha\beta]} = 0, \quad \delta\omega^\beta = 3\xi^\beta, \tag{2.8}$$

under the remaining gauge symmetry (2.4). Such symmetry allows us to set

$$\omega^\beta = 0, \tag{2.9}$$

thus reducing the Lagrangian (2.6) to

$$\mathcal{L}_Z = \frac{1}{2}\partial^\beta \bar{\omega}_{\mu[\alpha\beta]}\partial_\gamma \bar{\omega}^{\alpha[\mu\gamma]} + \frac{m^2}{2}\bar{\omega}^{\mu[\alpha\beta]}\bar{\omega}_{\alpha[\mu\beta]}. \tag{2.10}$$

To make contact with the CF Lagrangian we introduce now the field $T^{\rho[\mu\nu]}$ which is dual to $\bar{\omega}^{\beta[\mu\gamma]}$,

$$\bar{\omega}_{\rho[\alpha\beta]} = \epsilon_{\alpha\beta\mu\nu} T_\rho{}^{[\mu\nu]}. \tag{2.11}$$

The first property to remark is that the traceless condition upon $\bar{\omega}_{\rho[\alpha\beta]}$ leads to the cyclic identity of the dual field

$$T^{\rho[\mu\nu]} + T^{\nu[\rho\mu]} + T^{\mu[\nu\rho]} = 0, \tag{2.12}$$

characteristic of the CF field. In terms of this new variable the Lagrangian (2.10) becomes

$$\begin{aligned}\mathcal{L}_Z = {} & \partial^\beta T^{\alpha[\mu\nu]}\partial_\beta T_{\alpha[\mu\nu]} - 2\partial^\beta T^\nu \partial_\beta T_\nu - 2\partial_\mu T^{\alpha[\mu\nu]}\partial^\beta T_{\alpha[\beta\nu]} - \partial_\beta T^{\beta[\mu\nu]}\partial^\alpha T_{\alpha[\mu\nu]} \\ & - 4T_\nu \partial_\mu \partial_\beta T^{\beta[\nu\mu]} + 2\partial_\nu T^\nu \partial^\beta T_\beta . - m^2\left(T^{\alpha[\sigma\tau]}T_{\alpha[\sigma\tau]} - 2T^\sigma T_\sigma\right),\end{aligned} \tag{2.13}$$

with $T^\sigma = T_\alpha{}^{[\alpha\sigma]}$. Incorporating now the field strength

$$F_{\nu[\alpha\beta\gamma]} = \partial_\alpha T_{\nu[\beta\gamma]} + \partial_\beta T_{\nu[\gamma\alpha]} + \partial_\gamma T_{\nu[\alpha\beta]}, \tag{2.14}$$

corresponding to $T_{\nu[\beta\gamma]}$ we see that this Lagrangian is proportional to that of CF [1, 2]

$$L_Z = \frac{1}{3}\left(F_{\nu[\alpha\beta\gamma]}F^{\nu[\alpha\beta\gamma]} - 3F^\gamma{}_{[\alpha\beta\gamma]}F_\sigma{}^{[\alpha\beta\sigma]} - 3m^2\left(T^{\alpha[\sigma\tau]}T_{\alpha[\sigma\tau]} - 2T^\sigma T_\sigma\right)\right). \tag{2.15}$$

This establishes that the Zinoviev dual action, with the given gauge fixing, is in fact the CF action in four dimensions.

2.2 The Curtright-Freund action from the Zinoviev parent action

Using Zinoviev approach we should be able to identify the parent Lagrangian at the level of the relevant fields $\omega_{\alpha[\beta\gamma]}$ and $h_{\mu\nu}$ in order to compare with the approach of refs. [16, 17] and understand how the CF duality in four dimensions can be obtained from that approach. Notice that here $h_{\mu\nu}$ is not a symmetrical field.

To this end we start from the gauge invariant full parent Lagrangian given by eqs. (2.1) and (2.5) of ref. [19]

$$\begin{aligned}\mathcal{L}_{h,A,\phi,\omega,F,\pi} = {} & \frac{1}{2}\left(\omega^\gamma\omega_\gamma - \omega^{\beta[\alpha\gamma]}\omega_{\alpha[\beta\gamma]}\right) - \left(\omega_\tau{}^{[\nu\alpha]} + \delta^\alpha_\tau \omega_\rho{}^{[\rho\nu]} - \delta^\nu_\tau \omega_\mu{}^{[\mu\alpha]}\right)\partial_\nu h_\alpha{}^\tau \\ & - \frac{1}{4}F^{[\alpha\beta]}F_{[\alpha\beta]} + F^{[\mu\nu]}\partial_\mu A_\nu - \frac{1}{2}\pi^\alpha\pi_\alpha + \pi^\mu\partial_\mu\phi + \sqrt{3}m\pi^\mu A_\mu \\ & - \frac{m}{\sqrt{2}}F^{[\mu\nu]}h_{\mu\nu} - \sqrt{2}m\omega^\mu A_\mu + \sqrt{\frac{3}{2}}m^2 h\phi - m^2\phi^2 + \frac{m^2}{2}\left(h^{\alpha\beta}h_{\beta\alpha} - h^2\right),\end{aligned} \tag{2.16}$$

which has the following symmetries

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \kappa_{[\mu\nu]} - \frac{m}{\sqrt{2}}\eta_{\mu\nu}\lambda, \qquad \delta h = \partial_\mu \xi^\mu - 2\sqrt{2}m\lambda,$$

$$\delta\omega_\mu{}^{[\alpha\beta]} = \partial_\mu \kappa^{[\alpha\beta]} + \frac{m^2}{2}\left(\delta^\alpha_\mu \xi^\beta - \delta^\beta_\mu \xi^\alpha\right), \qquad \delta\omega_\mu{}^{[\mu\beta]} = \partial_\mu \kappa^{[\mu\beta]} + \frac{3}{2}m^2\xi^\beta,$$

$$\delta A_\mu = \frac{m}{\sqrt{2}}\xi_\mu + \partial_\mu \lambda, \qquad \delta F^{[\alpha\beta]} = -m\sqrt{2}\kappa^{[\alpha\beta]},$$

$$\delta\pi^\alpha = \sqrt{\frac{3}{2}}m^2\xi^\alpha, \qquad \delta\phi = -m\sqrt{3}\lambda. \tag{2.17}$$

Again, the corresponding items in ref. [19] are rewritten here in the metric $(-,+,+,+)$. The basic idea is to eliminate the auxiliary fields either by their equations of motion or by gauge fixing. The first step is the elimination of π_α via its equations of motion, which yield

$$\pi_\alpha = \partial_\alpha \phi + \sqrt{3}mA_\alpha, \tag{2.18}$$

leading to the remaining Lagrangian

$$\begin{aligned}\mathcal{L}_{h,A,\phi,\omega,F} = & \frac{1}{2}\left(\omega^\gamma\omega_\gamma - \omega^{\beta[\alpha\gamma]}\omega_{\alpha[\beta\gamma]}\right) - \left(\omega_\tau{}^{[\nu\alpha]} + \delta^\alpha_\tau \omega_\rho{}^{[\rho\nu]} - \delta^\nu_\tau \omega_\mu{}^{[\mu\alpha]}\right)\partial_\nu h_\alpha^\tau \\ & - \frac{1}{4}F^{[\alpha\beta]}F_{[\alpha\beta]} + F^{[\mu\nu]}\partial_\mu A_\nu + \frac{1}{2}\left(\partial_\alpha\phi + \sqrt{3}mA_\alpha\right)^2 + \sqrt{\frac{3}{2}}m^2 h\phi - m^2\phi^2 \\ & - \frac{m}{\sqrt{2}}F^{[\mu\nu]}h_{\mu\nu} - \sqrt{2}m\omega^\mu A_\mu + \frac{m^2}{2}\left(h^{\alpha\beta}h_{\beta\alpha} - h^2\right).\end{aligned} \tag{2.19}$$

The above Lagrangian is still invariant under the transformations (2.17), leaving out the transformation $\delta\pi^\alpha$. In this way the gauge freedom can be fixed using the parameters $\kappa^{[\alpha\beta]}$, ξ_μ and λ to set at zero the fields $F^{[\alpha\beta]}$, A_μ and ϕ respectively. Then we obtain

$$\mathcal{L}_{h,;\omega} = \frac{1}{2}\left(\omega^\gamma\omega_\gamma - \omega^{\beta[\alpha\gamma]}\omega_{\alpha[\beta\gamma]}\right) - \left(\omega^{\tau[\nu\alpha]} + \eta^{\tau\alpha}\omega^\nu - \eta^{\tau\nu}\omega^\alpha\right)\partial_\nu h_{\alpha\tau} + \frac{m^2}{2}\left(h^{\alpha\beta}h_{\beta\alpha} - h^2\right). \tag{2.20}$$

Observe that we still have a non-symmetrical $h_{\alpha\beta}$ with a specific choice for the FP mass term. Let us also remark that the Lagrangian (2.20) corresponds to the selection $a = 0$ in the parameter of the corresponding Lagrangian in the Introduction of ref. [19]. This is exhibited as a simple example of the ambiguities in the dual theory introduced by constructing the parent Lagrangian with arbitrary coefficients, restricted only by the condition that after eliminating the field $\omega_{\alpha[\beta\gamma]}$ the standard FP theory is recovered, as it is done in refs. [16, 17]. Nevertheless, given that we arrive at the condition $a = 0$ only after a very particular gauge fixing and field elimination via equations of motion, we take this as an indication that these two generally non-commuting and non-unique processes will also introduce ambiguities in the final Zinoviev Lagrangian containing only the dual propagating field. A more detailed discussion of this point is given at the end of section IV.

The above Lagrangian (2.20) can be written in terms of the massless West parent Lagrangian [12], with well known duality properties. To this end we introduce the following field transformation

$$Y^{\tau[\alpha\nu]} = \omega^{\tau[\nu\alpha]} + \eta^{\tau\alpha}\omega^\nu - \eta^{\tau\nu}\omega^\alpha, \tag{2.21}$$

in such a way that

$$Y^{\alpha} = -2\omega^{\alpha}, \tag{2.22}$$

with $Y^{\alpha} = \eta_{\tau\nu} Y^{\tau[\alpha\nu]}$. This transformation can be inverted as

$$\omega_{\alpha[\beta\gamma]} = Y_{\alpha[\gamma\beta]} + \frac{1}{2}\eta_{\alpha\gamma} Y_{\beta} - \frac{1}{2}\eta_{\alpha\beta} Y_{\gamma}. \tag{2.23}$$

Applying this transformation to the Lagrangian (2.20) we finally obtain

$$\mathcal{L}_{h;Y} = \frac{1}{2}\left[Y^{\tau[\nu\alpha]}\left(\partial_{\nu} h_{\alpha\tau} - \partial_{\alpha} h_{\nu\tau}\right) - Y^{\beta[\gamma\alpha]} Y_{\alpha[\gamma\beta]} + \frac{1}{2} Y^{\alpha} Y_{\alpha} + m^2\left(h^{\alpha\beta} h_{\beta\alpha} - h^2\right)\right]. \tag{2.24}$$

This is precisely the West action plus a FP type mass term in the notation of ref. [10]. As shown in this reference, the above Lagrangian in the massless case leads to the FP one after $Y^{\beta[\alpha\gamma]}$ is eliminated using the corresponding equations of motion. The massive case is completely analogous because the equations of motion for $Y_{\alpha[\beta\gamma]}$ do not involve the mass term. Thus, the kinetic energy piece of the action in terms of $h_{\alpha\beta}$ involves the antisymmetric part $h_{[\alpha\beta]}$ only as a total derivative. The mass term contributes with a term proportional to $h_{[\alpha\beta]} h^{[\alpha\beta]}$ which leads to the equation of motion $h_{[\alpha\beta]} = 0$. It is rather remarkable that the FP formulation is recovered despite the fact that $h_{\alpha\beta}$ is non-symmetrical. The above parent action is not a particular case of those employed in refs. [16, 17], where it was assumed that $h_{\alpha\beta} = h_{\beta\alpha}$ from the very beginning. Let us recall that the CF case was not obtained in such references.

We will now show, from this point of view, that the dual theory corresponds precisely to the CF Lagrangian by explicitly eliminating $h_{\alpha\beta}$ from the parent Lagrangian (2.24). The equations of motion for $h_{\alpha\beta}$ give

$$h^{\alpha\beta} = \frac{1}{m^2}\left(\partial_{\nu} Y^{\alpha[\nu\beta]} - \frac{1}{3}\eta^{\beta\alpha}\partial_{\nu} Y^{\nu}\right), \qquad h = -\frac{1}{3m^2}\partial_{\nu} Y^{\nu}. \tag{2.25}$$

After the substitutions (2.25) are made in (2.24), the final rescaled Lagrangian is

$$\tilde{\mathcal{L}}_Y = -2m^2 \mathcal{L}_Y = \partial_{\nu} Y^{\alpha[\nu\beta]} \partial^{\rho} Y_{\beta[\rho\alpha]} - \frac{1}{3}\left(\partial_{\nu} Y^{\nu}\right)^2 + m^2\left[Y^{\beta[\gamma\alpha]} Y_{\alpha[\gamma\beta]} - \frac{1}{2} Y^{\alpha} Y_{\alpha}\right]. \tag{2.26}$$

In order to make contact with the Lagrangian (2.10) which, as shown in the previous subsection, leads directly to the CF action we still need to introduce the traceless field $\bar{\omega}_{\rho[\alpha\beta]}$ according to eq. (2.7). In this way the final change of variables turns out to be

$$Y^{\tau[\alpha\nu]} = \bar{\omega}^{\tau[\nu\alpha]} + \frac{2}{3}\left(\eta^{\tau\alpha}\omega^{\nu} - \eta^{\tau\nu}\omega^{\alpha}\right). \tag{2.27}$$

After substituting in the Lagrangian (2.26) we obtain the dual one

$$\tilde{\mathcal{L}}_{\bar{\omega}} = \frac{1}{2}\partial_{\nu}\bar{\omega}^{\alpha[\beta\nu]}\partial^{\rho}\bar{\omega}_{\beta[\alpha\rho]} + \frac{m^2}{2}\left[\bar{\omega}^{\beta[\alpha\gamma]}\bar{\omega}_{\alpha[\beta\gamma]} - \frac{2}{3}\omega^{\alpha}\omega_{\alpha}\right]. \tag{2.28}$$

In fact, the term $(\partial_{\nu}\omega^{\nu})^2$ cancels out in the kinetic piece of Lagrangian (2.26), while contributions proportional to $\omega^{\nu}\omega_{\nu}$ in the mass term lead to $\omega_{\nu} = 0$ by the equations of motion. Finally, the Lagrangian (2.28) is identical to (2.10), thus leading to the CF final action.

This establishes that the parent Lagrangian of Zinoviev, with the above specific gauge fixing, gives a duality relation between the FP and the CF actions for a massive spin two field in four dimensions.

We emphasize that the above duality relation can not be obtained with the formulation presented in refs. [16, 17]. The reason is that there the tensor $h_{\mu\nu}$ in the parent Lagrangian is taken as symmetric, while in the parent Lagrangian of Zinoviev it has no definite symmetry. If we eliminate the field $Y^{\tau[\nu\alpha]}$ both Lagrangians lead to the FP one with $h_{\mu\nu}$ symmetric, but this difference is crucial when the eliminated field is $h_{\mu\nu}$, as can easily be visualized as follows. If we introduce the decomposition

$$h_{\alpha\tau} = h_{\{\alpha\tau\}} + h_{[\alpha\tau]}, \tag{2.29}$$

in the Lagrangian (2.24),where $h_{\mu\nu}$ is not symmetrical, we get

$$\begin{aligned}\mathcal{L}_{h;Y} = & -\tilde{Y}^{\nu[\alpha\tau]}\partial_\nu h_{[\alpha\tau]} - \frac{m^2}{2}h^{[\alpha\tau]}h_{[\alpha\tau]} \\ & + \tilde{Y}^{\nu\{\alpha\tau\}}\partial_\nu h_{\{\alpha\tau\}} - \frac{1}{2}\left(\tilde{Y}^{\nu\{\alpha\tau\}}\tilde{Y}_{\nu\{\alpha\tau\}} - \tilde{Y}^{\nu[\alpha\tau]}\tilde{Y}_{\nu[\alpha\tau]}\right) \\ & + \frac{1}{4}Y^\alpha Y_\alpha + \frac{m^2}{2}\left(h^{\{\alpha\tau\}}h_{\{\alpha\tau\}} - h^2\right),\end{aligned} \tag{2.30}$$

where

$$\tilde{Y}^{\nu[\tau\alpha]} = \frac{1}{2}\left(Y^{\tau[\nu\alpha]} - Y^{\alpha[\nu\tau]}\right), \quad \tilde{Y}^{\nu\{\tau\alpha\}} = \frac{1}{2}\left(Y^{\tau[\nu\alpha]} + Y^{\alpha[\nu\tau]}\right), \quad Y^\alpha = \eta_{\tau\nu}\tilde{Y}^{\alpha\{\tau\nu\}}. \tag{2.31}$$

Now we can eliminate $h_{[\alpha\tau]}$ using its equation of motion

$$h^{[\alpha\tau]} = \frac{1}{m^2}\partial_\nu\tilde{Y}^{\nu[\alpha\tau]}, \tag{2.32}$$

obtaining a Lagrangian that only contains $h_{\{\alpha\tau\}}$

$$\begin{aligned}\mathcal{L}_{h;Y} = & \frac{1}{2m^2}\partial_\nu\tilde{Y}^{\nu[\alpha\tau]}\partial^\mu\tilde{Y}_{\mu[\alpha\tau]} + \frac{1}{2}\tilde{Y}^{\nu[\alpha\tau]}\tilde{Y}_{\nu[\alpha\tau]} \\ & + \tilde{Y}^{\nu\{\alpha\tau\}}\partial_\nu h_{\{\alpha\tau\}} - \frac{1}{2}\tilde{Y}^{\nu\{\alpha\tau\}}\tilde{Y}_{\nu\{\alpha\tau\}} + \frac{1}{4}Y^\alpha Y_\alpha + \frac{m^2}{2}\left(h^{\{\alpha\tau\}}h_{\{\alpha\tau\}} - h^2\right).\end{aligned} \tag{2.33}$$

If, on the other hand, we consider the Lagrangian (2.24) with $h_{\alpha\tau}$ purely symmetric its elimination leads to

$$\begin{aligned}\mathcal{L}_{h;Y} = & \frac{1}{2}\tilde{Y}^{\nu[\alpha\tau]}\tilde{Y}_{\nu[\alpha\tau]} \\ & + \tilde{Y}^{\nu\{\alpha\tau\}}\partial_\nu h_{\{\alpha\tau\}} - \frac{1}{2}\tilde{Y}^{\nu\{\alpha\tau\}}\tilde{Y}_{\nu\{\alpha\tau\}} + \frac{1}{4}Y^\alpha Y_\alpha + \frac{m^2}{2}\left(h^{\{\alpha\tau\}}h_{\{\alpha\tau\}} - h^2\right).\end{aligned} \tag{2.34}$$

It is clear that in the first case the field $\tilde{Y}^{\nu[\tau\alpha]}$ is a dynamical one, while in the second one it is null. This states the difference between the dual theories generated in each case, and gives us the clue to modify the approach of refs. [16, 17] to generate a duality transformation that connects the FP theory with a Curtright-type formulation in arbitrary dimensions.

3. The parent action and the dual formulation in arbitrary dimensions

Our aim is the construction of a dual description to the FP formulation for a massive spin two field $h_{AB} = h_{BA}$ in arbitrary dimensions. We can follow the procedure developed in refs. [16, 17] to construct first order parent Lagrangians, but now starting with a nonsymmetric field h_{AB}. According to the discussion there presented, it is possible to construct several families of dual theories. These parent Lagrangians can be generalized to arbitrary dimensions. Nevertheless, to be specific, in this work we will consider only the dual theory generated by a parent Lagrangian that has the form of the one introduced in ref. [12] and discussed in ref. [10], which corresponds to the Vasiliev description for a massless spin two, plus the modified FP mass term proposed by Zinoviev. This is a generalization to unsymmetrical h_{AB} of a special case of the families just mentioned, and we defer a detailed study of the general situation in arbitrary dimensions for future work. Thus, in a flat D-dimensional space-time with metric $diag(-++++ +, \ldots, +)$ we take

$$S = \frac{1}{2}\int d^D x \left[Y^{C[AB]}\left(\partial_A h_{BC} - \partial_B h_{AC}\right) - Y_{C[AB]} Y^{B[AC]} + \frac{1}{(D-2)} Y^B{}_{[AB]} Y_C{}^{[AC]} \right.$$
$$\left. + m^2\left(h_{AB}h^{BA} - h^2\right)\right], \tag{3.1}$$

as our parent action. Here the fields are h_{BC} and $Y^{C[AB]}$, with D^2 and $D^2(D-1)/2$ independent components respectively. Redefining $Y^{C[AB]} \to -Y^{C[AB]}/\sqrt{2}$ and $h_{AB} \to \sqrt{2} h_{AB}$ this action becomes the action (4.15) of ref. [12] plus a FP mass term, up to a global minus sign.

The derivation of the FP action starting from the action (3.1) is the same as in ref. [10], because one needs to solve for $Y^{C[AB]}$, which does not involve the additional mass term. We only write the solution in our slightly modified conventions. The resulting expression for $Y_{B[AC]}$ in terms of h_{AB} is:

$$Y_{B[AC]} = \frac{1}{2}\left[\partial_A\left(h_{BC} + h_{CB}\right) - \partial_C\left(h_{BA} + h_{AB}\right) - \partial_B\left(h_{AC} - h_{CA}\right)\right]$$
$$+ \eta_{BC}\left(\partial^D h_{AD} - \partial_A h\right) - \eta_{BA}\left(\partial^D h_{CD} - \partial_C h\right), \tag{3.2}$$

$$Y_A = Y_{B[AC]}\eta^{\{CB\}} = -(D-2)\left(\partial_A h - \partial^B h_{AB}\right). \tag{3.3}$$

These expressions allow us to eliminate this field in the action (3.1). Splitting h^{AB} in its symmetric and antisymmetric parts

$$h^{AB} = h^{\{AB\}} + h^{[AB]}, \tag{3.4}$$

and dropping total derivatives we get

$$S = \frac{1}{2}\int d^D x \left[-\partial_A h_{\{BC\}}\partial^A h^{\{CB\}} + 2\partial^B h_{\{BC\}}\partial_A h^{\{AC\}} - 2\partial_A h \partial_E h^{\{AE\}} + \partial_A h \partial^A h \right.$$
$$\left. - m^2\left(h_{\{AB\}}h^{\{BA\}} + h_{[AB]}h^{[BA]} - h^2\right)\right]. \tag{3.5}$$

By using the Euler-Lagrange equation of $h_{[AB]}$ we get $h_{[AB]} = 0$, and thus we finally obtain

$$S = \frac{1}{2}\int d^Dx \left[-\partial_A h_{\{BC\}}\partial^A h^{\{CB\}} + 2\partial^B h_{\{BC\}}\partial_A h^{\{AC\}} - 2\partial_A h \partial_E h^{\{AE\}} + \partial_A h \partial^A h \right.$$
$$\left. -m^2 \left(h_{\{AB\}} h^{\{BA\}} - h^2 \right) \right], \tag{3.6}$$

which is precisely the massive FP action in D dimensions. The Euler-Lagrange equations yield $(D+1)$ constraints, $\partial_A h^{\{AB\}} = 0$ and $h_A{}^A = 0$, and thus the number of degrees of freedom is

$$\mathcal{F}_m^D = \frac{D}{2}(D+1) - (D+1) = \frac{D}{2}(D-1) - 1. \tag{3.7}$$

To obtain the dual description we eliminate h^{AB} using its corresponding equations of motion obtained from the action (3.1), which yield

$$h^{AB} = \frac{1}{m^2}\left(\frac{1}{D-1}\eta^{AB}\partial_C Y^C - \partial_C Y^{A[CB]} \right), \tag{3.8}$$

leading to the following action for $Y_{C[AB]}$

$$m^2 S = \int d^Dx \left[\partial_A Y^{C[AB]}\partial^E Y_{B[EC]} - \frac{1}{D-1}(\partial_A Y^A)^2 \right.$$
$$\left. + m^2 \left(Y_{C[AB]} Y^{B[AC]} - \frac{1}{D-2} Y_A Y^A \right) \right]. \tag{3.9}$$

To compare with the usual formulation of the Curtright Lagrangian it is useful to introduce the change of variables

$$Y^{C[AB]} = \bar{w}^{C[AB]} + \frac{1}{(D-1)}(\eta^{CB}Y^A - \eta^{CA}Y^B), \tag{3.10}$$

where $\bar{w}^{C[AB]}$ has a null trace, $\bar{w}_A{}^{[AB]} = 0$. Rescaling the action to absorb the m^2 factor we obtain

$$S = \frac{1}{2}\int d^Dx \left[\partial_A \bar{w}^{C[BA]}\partial^E \bar{w}_{B[CE]} + m^2 \left(\bar{w}^{C[AB]}\bar{w}_{B[AC]} - \frac{1}{(D-1)(D-2)} Y^A Y_A \right) \right], \tag{3.11}$$

which clearly shows that the trace of $Y^{C[BA]}$ is an irrelevant variable that can be eliminated from the Lagrangian using its equation of motion. Thus we finally get

$$S = \int d^Dx \frac{1}{2}\left[\partial_A \bar{w}^{C[BA]}\partial^E \bar{w}_{B[CE]} + m^2 \bar{w}^{C[AB]}\bar{w}_{B[AC]} \right]. \tag{3.12}$$

This is the generalization to arbitrary dimensions of the Lagrangian (2.10).

The derivative term has the gauge symmetries

$$\delta \bar{w}_C{}^{[AB]} = \epsilon^{ABM_1M_2M_3\ldots M_{D-2}}\partial_{M_1} S_{\{CM_2\}[M_3\ldots M_{D-2}]}, \tag{3.13}$$
$$\delta \bar{w}_C{}^{[AB]} = \epsilon^{ABM_1M_2M_3\ldots M_{D-2}}\left(\partial_{M_1} A_{[CM_2M_3\ldots M_{D-2}]} + \partial_C A_{[M_1M_2\ldots M_{D-2}]} \right). \tag{3.14}$$

The mass term breaks these symmetries and assigns to the true degrees of freedom a mass m.

In order to make contact with the usual expression for the Curtright Langrangian in $D = 4$, where the basic field satisfies a cyclic condition, we need to introduce the Hodge-dual of $\bar{w}^{C[AB]}$

$$T_{P[Q_1Q_2...Q_{D-2}]} = \frac{1}{2}\bar{w}_P{}^{[AB]}\,\epsilon_{ABQ_1Q_2...Q_{D-2}}, \tag{3.15}$$

which is a dimension-dependent tensor of rank $(D-1)$ completely antisymmetric in its last $(D-2)$ indices. The resulting action corresponding to the field $T_{P[Q_1Q_2...Q_{D-2}]}$ will be taken as the dual version of the original FP formulation. We can invert eq. (3.15) obtaining

$$\bar{w}_C{}^{[AB]} = -\frac{1}{(D-2)!}T_{C[Q_1Q_2...Q_{D-2}]}\epsilon^{Q_1Q_2...Q_{D-2}AB}. \tag{3.16}$$

Here we are using the basic definition

$$\epsilon^{A_1A_2...A_{D-1}A_D}\epsilon_{B_1B_2...B_{D-1}B_D} = -\delta^{[A_1A_2...A_{N-1}A_N]}_{[B_1B_2...B_{N-1}B_N]}, \tag{3.17}$$

where the required properties of the fully antisymmetrized Kronecker delta $\delta^{[A_1A_2...A_{N-1}A_N]}_{[B_1B_2...B_{N-1}B_N]}$, $N \leq D$, together with its contraction with some relevant tensors, are written down in the appendix A. There we have included all the cases relevant to the calculation and we will not specify the particular relation used in any of the following steps. The traceless condition upon $\bar{w}_{A[BC]}$ leads to the cyclic identity for the dual field

$$\epsilon^{Q_1Q_2...Q_{D-2}AS}T_{S[Q_1Q_2...Q_{D-2}]} = 0. \tag{3.18}$$

It is convenient to introduce the field strength $F^{A[Q_1Q_2...Q_{D-2}Q_{D-1}]}$, which is a tensor of rank D, associated with the potential $T^{A[Q_1Q_2...Q_{D-2}]}$ given by

$$F^{A[Q_1Q_2...Q_{D-2}Q_{D-1}]} = \frac{1}{(D-2)!}\delta^{[Q_1Q_2...Q_{D-2}Q_{D-1}]}_{[A_1A_2...A_{D-2}A_{D-1}]}\partial^{A_1}T^{A[A_2...A_{D-2}A_{D-1}]}. \tag{3.19}$$

In this way $F^{A[Q_1Q_2...Q_{D-2}Q_{D-1}]}$ satisfies

$$\epsilon_{CQ_1Q_2...Q_{D-2}B}\,F^{A[BQ_1Q_2...Q_{D-2}]} = (D-1)\,\epsilon_{CQ_1Q_2...Q_{D-2}B}\partial^BT^{A[Q_1Q_2...Q_{D-2}]}. \tag{3.20}$$

In terms of the Hodge-dual the kinetic part of the Lagrangian becomes

$$\begin{aligned}\partial_A\bar{w}^{C[BA]}\partial^E\bar{w}_{B[CE]} = -\frac{1}{(D-2)!}\Bigg[&\frac{1}{(D-1)}F_B{}^{[AQ_1Q_2...Q_{D-2}]}\,F^B{}_{[AQ_1Q_2...Q_{D-2}]} \\ &- F_A{}^{[AQ_1Q_2...Q_{D-2}]}F^B{}_{[BQ_1Q_2...Q_{D-2}]}\Bigg],\end{aligned} \tag{3.21}$$

while the mass terms acquires the form

$$\begin{aligned}\bar{w}^{C[AB]}\bar{w}_{B[AC]} = -\frac{1}{(D-2)!}\Bigg[&T_{B[Q_1Q_2...Q_{D-2}]}T^{B[Q_1Q_2...Q_{D-2}]} \\ &-(D-2)T^C{}_{[CQ_2...Q_{D-3}]}T_B{}^{[BQ_2...Q_{D-3}]}\Bigg].\end{aligned} \tag{3.22}$$

Thus, the final action dual to FP in arbitrary dimensions can be written

$$S(T) = \int d^D x \Big\{ - \Big[\frac{1}{(D-1)} F_B{}^{[AQ_1 \ldots Q_{D-2}]} F^B{}_{[AQ_1 \ldots Q_{D-2}]} - F_A{}^{[AQ_1 \ldots Q_{D-2}]} F^B{}_{[BQ_1 \ldots Q_{D-2}]} \Big] \\ - m^2 \Big[T_{B[Q_1 \ldots Q_{D-2}]} T^{B[Q_1 \ldots Q_{D-2}]} - (D-2) T^C{}_{[CQ_2 \ldots Q_{D-3}]} T_B{}^{[BQ_2 \ldots Q_{D-3}]} \Big] \Big\}, \tag{3.23}$$

after an adequate rescaling of the original action. Here the field $T_{B[Q_1 Q_2 \ldots Q_{D-2}]}$ satisfies the cyclic condition (3.18), and the gauge symmetries of the kinetic terms, broken by the mass term, now become (up to global numerical factors)

$$\delta T_{P[Q_1 Q_2 \ldots Q_{D-2}]} = \delta^{[M_1 M_2 \ldots M_{D-2}]}_{[Q_1 Q_2 \ldots Q_{D-2}]} \partial_{M_1} S_{\{PM_2\}[M_3 M_4 \ldots M_{D-2}]}, \tag{3.24}$$

$$\delta T_{P[Q_1 Q_2 \ldots Q_{D-2}]} = \frac{1}{(D-2)!} \delta^{[M_1 M_2 \ldots M_{D-2}]}_{[Q_1 Q_2 \ldots Q_{D-2}]} \partial_{M_1} A_{[PM_2 \ldots M_{D-2}]} + \partial_P A_{[M_1 M_2 \ldots M_{D-2}]}. \tag{3.25}$$

The action (3.23), which is dual to FP in arbitrary dimensions and which is free from auxiliary fields, is the main result of this paper. It reduces to the CF action in four dimensions. We observe that the dual Lagrangians (2.10) and (3.12) have identical form when written in terms of the traceless field $\bar{w}^{C[AB]}$. Nevertheless this is not the case after the introduction of the dual field of $\bar{w}^{C[AB]}$ which will satisfy the cyclic identity.

The action (3.23) leads to the equation of motion

$$\Big[\delta^{[AQ_2 \ldots Q_{D-1}]}_{[A_1 A_2 \ldots A_{D-1}]} \delta^B_C - (D-1)\, \delta^{[BQ_2 \ldots Q_{D-1}]}_{[A_1 A_2 \ldots A_{D-1}]} \delta^A_C \Big] \partial^{A_1} F^C{}_{[AQ_2 \ldots Q_{D-1}]} \\ - m^2 (D-2)! \Big[T^B{}_{[A_2 \ldots A_{D-1}]} - \frac{1}{(D-3)!} \delta^{[BM_3 \ldots M_{D-1}]}_{[A_2 \ldots \ldots A_{D-1}]} T^C{}_{[CM_3 \ldots M_{D-1}]} \Big] = 0, \tag{3.26}$$

or more explicitly, in terms of the derivatives of the mixed symmetry tensor $T^B{}_{[A_2 \ldots A_{D-2} A_{D-1}]}$

$$\Big[(D-2)! \delta^{[M_1 \ldots M_{D-1}]}_{[A_1 \ldots A_{D-1}]} \delta^B_C - \delta^{[BQ_2 \ldots Q_{D-1}]}_{[A_1 A_2 \ldots A_{D-1}]} \delta^{[M_1 \ldots M_{D-1}]}_{[CQ_2 \ldots \, Q_{D-1}]} \Big] \partial^{A_1} \partial_{M_1} T^C{}_{[M_2 \ldots M_{D-1}]} \\ - m^2 [(D-2)!]^2 \Big[T^B{}_{[A_2 \ldots A_{D-1}]} - \frac{1}{(D-3)!} \delta^{[BM_3 \ldots M_{D-1}]}_{[A_2 \ldots \ldots A_{D-1}]} T^C{}_{[CM_3 \ldots M_{D-1}]} \Big] = 0. \tag{3.27}$$

In appendix B we derive the Lagrangian constraints arising from this equation of motion. The complete set of constraints which the dual field $T^B{}_{[A_2 \ldots A_{D-1}]}$ satisfies is

$$\epsilon^{Q_1 Q_2 \ldots Q_{D-2} AS} T_{S[Q_1 Q_2 \ldots Q_{D-2}]} = 0, \tag{3.28}$$

$$T^B{}_{[BA_3 \ldots A_{D-1}]} = 0, \tag{3.29}$$

$$\partial^D T^B{}_{[DA_3 \ldots A_{D-1}]} = 0, \tag{3.30}$$

$$\partial_B T^B{}_{[A_2 A_3 \ldots A_{D-1}]} = 0. \tag{3.31}$$

After implementing these constraints the equation of motion reduces to its simplest form

$$\left(\partial^2 - m^2 \right) T^B{}_{[A_2 \ldots A_{D-1}]} = 0. \tag{3.32}$$

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The field $T^B{}_{[A_2 \ldots A_{D-1}]}$ in D dimensions has $\mathcal{N} = D^2(D-1)/2$ independent components, but it must satisfy the constraints (3.28)–(3.31). To identify the degrees of freedom it is convenient to write these constraints in momentum space, and in the rest frame where $k_M = (m, 0, 0, \ldots\ldots, 0, 0, 0)$. In such a way the constraints (3.30) and (3.31) imply that only the components with purely spatial indices are non null, and give the independent constraints:

$$T^{I_2}{}_{[0 I_3 \ldots I_{D-1}]} = 0 \quad \rightarrow \quad (D-1)\,\frac{(D-1)!}{2!\,(D-3)!} \ \text{ constraints,} \tag{3.33}$$

$$T^{0}{}_{[I_2 I_3 \ldots I_{D-1}]} = 0 \quad \rightarrow \quad \frac{(D-1)!}{(D-2)!} \ \text{ constraints,} \tag{3.34}$$

$$T^{0}{}_{[0 I_3 \ldots I_{D-1}]} = 0 \quad \rightarrow \quad \frac{(D-1)!}{2!\,(D-3)!} \ \text{ constraints,} \tag{3.35}$$

where now the indices I_i run only on spatial values, $I_i = 1, 2, \ldots, D-1$. Up to this stage we have $(D-1)\left[D\,(D-2)+2\right]/2$ constraints. Taking the above relations into account, the cyclic identity (3.28) yields only one additional constraint corresponding to the choice $A = 0$ in the expression

$$\epsilon^{I_1 I_2 \ldots I_{D-2} A I} T_{I[I_1 I_2 \ldots I_{D-2}]} = 0. \tag{3.36}$$

Finally, the constraints (3.29) lead to

$$T^B{}_{[B I_3 \ldots I_{D-1}]} = 0\,, \qquad B, I_i = 1, \ldots, D-1, \ \rightarrow \ \frac{(D-1)!}{2(D-3)!} \ \text{ constraints.} \tag{3.37}$$

Thus the total number of constraints is

$$\mathcal{C} = \frac{1}{2} D\,(D-1)^2 + 1, \tag{3.38}$$

and the number of degrees of freedom actually is

$$\mathcal{G} = \mathcal{N} - \mathcal{C} = \frac{1}{2} D\,(D-1) - 1, \tag{3.39}$$

which indeed is the same number obtained in eq. (3.7) for $h_{\{AB\}}$ in the FP formulation.

4. Final comments

In this paper we have investigated the possibility of constructing dual theories for the massive gravitational field in arbitrary dimensions, following a generalization of the ideas originally proposed in refs. [2, 1]. In these works a dual relation between massive Fierz-Pauli and a third rank mixed symmetry tensor $T_{A[Q_1 Q_2]}$ was explored, failing in the attempt of constructing such a relation. The possibility of using higher rank tensors with mixed symmetry was also mentioned there, but this approach was not further developed. Thus, the problem of finding the appropriate parent action providing the duality between the Fierz-Pauli action and those for the mixed symmetry tensors proposed in refs. [2, 1] has remained an open question. In the present paper we have shown that such a dual relation can be obtained in four dimensions and we have also proposed a generalization to arbitrary dimensions in terms of a $(D-1)$-rank tensor $T_{A[Q_1Q_2\ldots Q_{D-2}]}$. The construction can also be presented in terms of the traceless field $\bar{\omega}_{A[BC]}$, dual to $T_{A[Q_1Q_2\ldots Q_{D-2}]}$, in terms of which the action has the same form in any dimension.

The motivation for our construction is rooted in the attempt to understand the relation between the Zinoviev approach [19], based on a first order parent Lagrangian having well defined gauge symmetries generated by a set of auxiliary fields, and the approach proposed in ref. [16], based on the most general form for a first order parent Lagrangian containing only the dual fields. In the Zinoviev formalism a duality transformation between Stueckelberg-like Lagrangians for massive fields is obtained, while in that of refs. [16, 17] the duality is directly stated at the level of the fields corresponding to different representations for the massive spin two degrees of freedom.

With the purpose of making contact between the two approaches, in section II we take as the starting point the first order parent Lagrangian (2.1) plus the terms (2.5) of ref. [19], in the flat space limit, which depends on the fields $\omega_\mu{}^{[\alpha\beta]}$, $h_\mu{}^\alpha$, $F^{[\alpha\beta]}$, A_μ, π^α, and ϕ. After eliminating π^α and being consistent with the remaining gauge symmetries, we use a gauge fixing such that all the auxiliary fields become null, and only the spin two dual fields $\omega_\mu{}^{[\alpha\beta]}$and $h_\mu{}^\alpha$ remain. From here, implementing an adequate transformation, we show that this gauge fixed parent Lagrangian is precisely equivalent to that proposed by West [12], plus a FP type mass term, in the notation of ref. [10]. This parent Lagrangian leads to massive Fierz-Pauli after eliminating $Y^{\beta[\alpha\gamma]}$. On the other hand, after eliminating $h_{\alpha\beta}$, we have shown that it is equivalent to the Curtright-Freund action in four dimensions. This establishes that in four dimensions the parent Lagrangian of Zinoviev with the above specific gauge fixing is equivalent to the West parent Lagrangian, which provides a duality relation between the Fierz-Pauli and the Curtright-Freund actions for a massive spin two field. We emphasize that the above duality relation between Fierz-Pauli and Curtright-Freund was not obtained in refs. [16, 17]. The reason is very simple: in such references the tensor $h_{\mu\nu}$ in the parent Lagrangian is taken as symmetric, while in the parent Lagrangian introduced by West it has no definite symmetry.

On the basis of the last observation, the formalism of refs. [16, 17] has been extended to arbitrary dimensions in section III, by replacing the symmetric $h_{\{AB\}}$ tensor in the particular parent Lagrangian (3.1) by one without a definite symmetry. In such a way we obtain a new description for the massive Fierz-Pauli gravitation in terms of a mixed symmetry tensor $T_{S[Q_1Q_2\ldots Q_{D-2}]}$, based on an action whose kinetic term satisfies the gauge symmetries compatible with the cyclic condition (3.18). We have also identified the propagating modes of this theory, showing that they correspond to purely transversal components of a traceless field. Within the parent Lagrangian formalism there are additional possibilities, starting from the general structure for the first order Lagrangian discussed in ref. [16] together with a nonsymmetric h_{AB}, which are not discussed here.

A comment regarding the two approaches considered in this work is now in order. Our parent Lagrangian construction is based on the most general first order Lagrangian that contains only a given field and its dual, provided that the elimination of the dual field yields the adequate theory for the original field. This most general Lagrangian may depend on several parameters, and thus the elimination of the original field leads to a multiparametric family of dual Lagrangians, i.e. for a given theory we can in general construct several dual descriptions. On the other hand, the Zinoviev approach is based on a different perspective, which leads to the construction of a Stueckelberg-type parent Lagrangian that contains the original and the dual variables together with a set of auxiliary fields required to implement certain gauge symmetries. To derive the dual Lagrangian in terms of the corresponding propagating physical field it is necessary not only to choose some necessary gauge fixings, but also to use some equations of motion. This can be readily appreciated in the four dimensional parent Lagrangian (2.1) and (2.5) of ref. [19], which starts with 55 independent fields plus 11 arbitrary functions to be gauge fixed. Going from the remaining 44 variables to the final 10 degrees of freedom requires either some field eliminations via equations of motion or some field redefinitions that unify certain combinations. Clearly this adds a lot of freedom to the final result. In this way, different gauge fixings will lead to dual Lagrangians which are equivalent from the point of view of belonging to the same gauge orbits of the original Lagrangian, but not necessarily equivalent among themselves, in the sense that they cannot be connected by modifying the actions with boundary terms. The alternative gauge fixed Lagrangians lead to different patterns for eliminating the remaining auxiliary variables by using their equations of motion. This opens up additional possibilities for the appearance of further non equivalent dual Lagrangians. The very different starting points of both approaches makes it very difficult, if at all possible, to establish a general relation between them. In this paper we have only shown that the Zinoviev approach with a given gauge fixing leads to a dual Lagrangian also contained in the first order parent Lagrangian approach. We defer for further work the study of the possible general connections between these two approaches.

In a nutshell we can summarize our results by saying that the parent Lagrangian looked for by Curtright and Freund for a massive spin two field in arbitrary dimensions is simply given by the Lagrangian of West [12] completed by the Fierz-Pauli mass term arising from the Zinoviev approach [19], and involving a non-symmetrical rank-two tensor.

A. Properties of the generalized antisymmetric Kronecker delta

We summarize some relations including the antisymmetrized generalized delta function together with its contractions with various antisymmetric tensors.

The completely antisymmetrized generalized delta function $\delta^{[A_1 A_2 \ldots. A_N]}_{[M_1 M_2 \ldots M_N]}$ in D dimensions having $N!$ terms ($N \leq D$), is defined as

$$\delta^{[A_1 A_2 \ldots. A_N]}_{[M_1 M_2 \ldots M_N]} = \det \begin{bmatrix} \delta^{A_1}_{M_1} & \delta^{A_2}_{M_1} & \ldots & \delta^{A_N}_{M_1} \\ \delta^{A_1}_{M_2} & \delta^{A_2}_{M_2} & \ldots & \delta^{A_N}_{M_2} \\ \ldots & \ldots & \ldots & \ldots \\ \delta^{A_1}_{M_N} & \delta^{A_2}_{M_N} & \ldots & \delta^{A_N}_{M_N} \end{bmatrix}, \tag{A.1}$$

having the basic decomposition property

$$\delta^{[A_1 \ldots. A_N]}_{[M_1 \ldots M_N]} = \sum_{I=1}^{N} (-1)^{(I-1)} \, \delta^{A_1}_{M_I} \, \delta^{[A_2 \ldots A_{I-1} A_I A_{I+1} \ldots. A_N]}_{[M_1 \ldots M_{I-1} \; M_{I+1} \ldots M_N]}. \tag{A.2}$$

One important property is the contraction of the first I indices

$$\delta^{[A_1\dots A_I A_{I+1}\dots A_N]}_{[A_1\dots A_I M_{I+1}\dots M_N]} = \frac{(D-N+I)!}{(D-N)!}\delta^{[A_{I+1}\dots A_D]}_{[M_{I+1}\dots M_D]}. \tag{A.3}$$

The following contractions follow directly from the definition

$$\delta^{[A_1\dots A_N]}_{[M_1\dots M_N]} S^{[M_1\dots M_N]} T_{[A_1\dots A_N]} = N!\ S^{[M_1\dots M_N]} T_{[M_1\dots M_N]}, \tag{A.4}$$

$$\begin{aligned} S^{M[M_1\dots M_{N-1}]} T_{A[A_1\dots A_{N-1}]}\ \delta^{[AA_1\dots A_{N-1}]}_{[MM_1\dots M_{N-1}]} = (N-1)!\ \Big[& S^{Q[M_1\dots M_{N-1}]} T_{Q[M_1\dots M_{N-1}]} \\ & - (N-1)\ S^{P[QM_1\dots M_{N-2}]} T_{Q[PM_1\dots M_{N-2}]} \Big], \end{aligned} \tag{A.5}$$

$$\begin{aligned} S^B{}_{[R_1\dots R_{N-1}]}\ T_A{}^{[Q_1\dots Q_{N-1}]}\ \delta^{[AR_1\dots R_{N-1}]}_{[BQ_1\dots Q_{N-1}]} = (N-1)!\ \Big[& S_A{}^{[Q_1\dots Q_{N-1}]}\ T^A{}_{[Q_1\dots Q_{N-1}]} \\ & -(N-1)\ S_A{}^{[AQ_1\dots Q_{N-2}]}\ T^B{}_{[BQ_1\dots Q_{N-2}]} \Big]. \end{aligned} \tag{A.6}$$

B. The lagrangian constraints on $T_{A[Q_1 Q_2\dots Q_{D-2}]}$

Starting from the equations of motion (3.27) we derive the constraints (3.29), (3.30) and (3.31), which together with the cyclic identity (3.28) provide the correct number of degrees of freedom for the dual field $T^B{}_{[DA_3\dots A_{D-1}]}$.

Contracting a derivative with one of the antisymmetric indices in the equation of motion (3.27) we obtain the following first set of constraints

$$\partial^D T^B{}_{[DA_3\dots A_{D-1}]} = \frac{1}{(D-3)!}\delta^{[BM_3\dots M_{D-1}]}_{[DA_3\dots A_{D-1}]}\partial^D T^C{}_{[CM_3\dots M_{D-1}]}. \tag{B.1}$$

Contracting next one of the antisymmetric free indices in (3.27) , for example A_2, with B we get

$$\begin{aligned} &(D-2)!\delta^{[M_1 M_2\dots M_{D-1}]}_{[BA_1 A_3\dots A_{D-1}]}\partial^{A_1}\partial_{M_1} T^B{}_{[M_2 M_3\dots M_{D-1}]} \\ &\quad - \delta^{[BQ_2 Q_3\dots Q_{D-1}]}_{[BA_1 A_3\dots A_{D-1}]}\delta^{[M_1 M_2\dots M_{D-1}]}_{[CQ_2\dots Q_{D-1}]}\partial^{A_1}\partial_{M_1} T^C{}_{[M_2 M_3\dots M_{D-1}]} \\ &\qquad + m^2\left[(D-2)!\right]^2 \left[T^B{}_{[BA_3\dots A_{D-1}]} - \frac{1}{(D-3)!}\delta^{[BM_3\dots M_{D-1}]}_{[BA_3\dots A_{D-1}]} T^C{}_{[CM_3\dots M_{D-1}]} \right] = 0. \end{aligned} \tag{B.2}$$

In fact the second term with derivatives is proportional to the first one. This can be proved using an adequate expansion of the antisymmetric delta according to (A.2). In this way the first derivative term can be written

$$\begin{aligned} &\delta^{[M_1 M_2\dots M_{D-1}]}_{[BA_1 A_3\dots A_{D-1}]}\partial^{A_1}\partial_{M_1} T^B{}_{[M_2\dots M_{D-1}]} \\ &\quad = \delta^{[M_2 M_3\dots M_{D-1}]}_{[A_1 A_3\dots A_{D-1}]}\left(\partial^{A_1}\partial_B T^B{}_{[M_2 M_3\dots M_{D-1}]} - (D-2)\partial^{A_1}\partial_{M_2} T^B{}_{[B\dots M_{D-1}]}\right) \end{aligned} \tag{B.3}$$

Using the relations (A.2) and (A.3), the second derivative term yields

$$\delta^{[BQ_2Q_3\ldots Q_{D-1}]}_{[BA_1A_3\ldots A_{D-1}]}\delta^{[M_1M_2\ldots M_{D-1}]}_{[CQ_2\ldots \quad Q_{D-1}]}\partial^{A_1}\partial_{M_1}T^{C}{}_{[M_2\ldots M_{D-1}]} =$$
$$= (D-2)!\delta^{[BQ_2Q_3\ldots Q_{D-1}]}_{[BA_1A_3\ldots A_{D-1}]}\left(\partial^{A_1}\partial_{C}T^{C}{}_{[Q_2Q_3\ldots Q_{D-1}]} - (D-2)\partial^{A_1}\partial_{Q_2}T^{C}{}_{[CQ_3\ldots Q_{D-1}]}\right)$$
$$= 2\,(D-2)!\delta^{[M_1M_2\quad\ldots M_{D-1}]}_{[BA_1A_3\ldots A_{D-1}]}\partial^{A_1}\partial_{M_1}T^{B}{}_{[M_2\ldots M_{D-1}]}. \tag{B.4}$$

and eq. (B.2) becomes

$$\delta^{[M_1M_2\ldots M_{D-1}]}_{[BA_1\ldots \ A_{D-1}]}\partial^{A_1}\partial_{M_1}T^{B}{}_{[M_2\ldots M_{D-1}]} + 2\,(D-2)!m^2\left[T^{C}{}_{[CM_3\ldots M_{D-1}]}\right] = 0. \tag{B.5}$$

Here we have used eq. (A.3) in order to rewrite the square bracket proportional to m^2 in eq. (B.2). The derivative term in eq. (B.5) can also be written

$$\delta^{[M_1\ldots M_{D-1}]}_{[BA_1\ldots A_{D-1}]}\partial^{A_1}\partial_{M_1}T^{B}{}_{[M_2\ldots M_{D-1}]} = (D-2)\Big[(D-3)!\partial^{A_1}\partial_{B}T^{B}{}_{[A_1A_3\ldots A_{D-1}]}$$
$$-\delta^{[M_2M_3\ldots M_{D-1}]}_{[A_1A_3\ldots A_{D-1}]}\partial^{A_1}\partial_{M_2}T^{B}{}_{[BM_3\ldots M_{D-1}]}\Big] \tag{B.6}$$

and using the first set of constraints already obtained, (B.1), we have

$$\delta^{[M_2\ldots M_{D-1}]}_{[A_1A_3\ldots A_{D-1}]}\partial^{A_1}\partial_{M_2}T^{B}{}_{[B\ldots M_{D-1}]} = (D-3)!\partial_{B}\partial^{D}T^{B}{}_{[DA_3\ldots A_{D-1}]}, \tag{B.7}$$

which finally yields

$$\delta^{[M_1M_2\ldots M_{D-1}]}_{[BA_1A_3\ldots A_{D-1}]}\partial^{A_1}\partial_{M_1}T^{B}{}_{[M_2\ldots M_{D-1}]} = 0. \tag{B.8}$$

In such a way eq. (B.5) reduces to a second set of constraints

$$T^{B}{}_{[BA_3\ldots A_{D-1}]} = 0. \tag{B.9}$$

Combining the constraints (B.1) and (B.9), together with (3.28) we have the following set of constraints for $T_{S[Q_1Q_2\ldots Q_{D-2}]}$

$$T^{B}{}_{[BA_3\ldots A_{D-1}]} = 0, \tag{B.10}$$
$$\partial^{D}T^{B}{}_{[DA_3\ldots A_{D-1}]} = 0. \tag{B.11}$$

The last set of constraints

$$\partial_{B}T^{B}{}_{[A_2A_3\ldots A_{D-1}]} = 0 \tag{B.12}$$

is obtained by explicitly rewriting the cyclic identity (3.28) and subsequently contracting a derivative with the unsymmetrized index in the first term. This contraction will appear among the antisymmetric indices in the remaining terms of the sum, each of which will be identically zero in virtue of the constraints (3.30).

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